# Filter Design

Table . Transformations.

|  |  |  |
| --- | --- | --- |
| Transformation | Transfer function |  |
| Change the gain at cut-off frequency |  |  |
| Shift the cut-off frequency |  |  |
| Invert the magnitude response |  |  |
| Lowpass to low shelf | ?? |  |
| Lowpass to bandpass |  |  |

Table . Transfer functions of common first and second order filters, and their equivalent forms based on simpler filters.

|  |  |  |
| --- | --- | --- |
|  | Transfer function | Steps |
| Prototype |  |  |
| 1st order low pass |  | Prototype 🡪 Change gain at cut-off 🡪 Shift cut-off frequency |
| 1st order low shelf |  | Prototype 🡪 Change gain at cut-off 🡪Create shelf🡪 Shift cut-off frequency |

### Derivation:

We can construct a low shelving filter by transforming our prototype filter, such that the square magnitude response is transformed from *H*2 to (*G*2-1)*H*2­+1. This transformation changes the extreme square magnitudes 0 and 1 of a low pass design to 1 and *G*2.

Recall that the poles will push up the magnitude response for nearby frequencies, and the zeros will pull it down. For the low shelving filter, we want to keep the poles where they are, thus retaining the transition at the crossover frequency. But now we shift the zeros to give the low shelving filter behaviour. Suppose the first order section of a prototype low pass filter is written as

Then the first order section of the low shelving filter becomes,

So .

We don’t change the gain at this crossover frequency.

Consider the prototype

,

Let’s first change the gain at the cut-off frequency to g=(*G*-1)/(*G*2-1). That way, when the square magnitude response is transformed from *H*2 to (*G*2-1)*H*2­+1, this gain will map to *G.*

To change this gain, apply this transformation.

,

,

,

,

,

=,

By plugging in the formula for alpha, and a lot of maths, we see that this reduces to

,

Now apply this shelving filter transformation,

,

=G

->?

Now we just need to shift this crossover frequency from p/2 to some . Use

### Check this:

Square magnitude

Since . This gives